

BC 3

Alternating Series

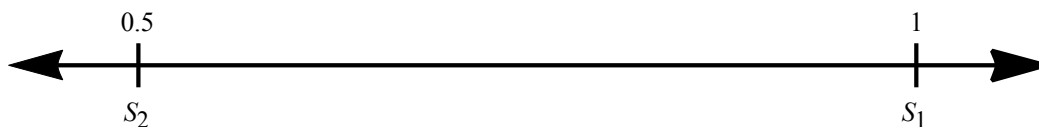
Name: _____

Defn: A series $\sum a_k$ is an alternating series if, with $a_n > 0$, it has the form

$$a_1 - a_2 + a_3 - a_4 + \dots \quad \text{OR} \quad -a_1 + a_2 - a_3 + a_4 - \dots$$

(1) Consider $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$, the alternating harmonic series.

Find $S_1, S_2, S_3, \dots, S_{10}$ to two decimal places, and plot each of these on the line below. (This has been started for you.)



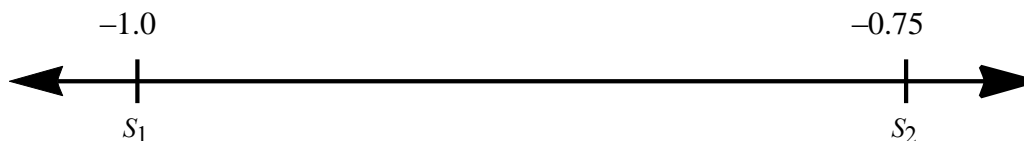
Describe the pattern of the S_n for even and odd values of n .

Based on this pattern, do you think the series converges?

Where is S ? (Write an inequality involving S using numbers you calculated above.)

(2) Consider the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = -1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \dots$

Find $S_1, S_2, S_3, \dots, S_{10}$ to three decimal places, and plot each on the line below. (Do not try to plot these precisely to scale!)



Describe the pattern of the S_n .

How does this differ from the pattern in the first example?

Do you think this series converges?

Where is S ? (Write an inequality involving S .)

It is true that both of the preceding series converge since the terms of both followed certain patterns. For a series $\sum a_k$, what are these conditions that will guarantee convergence? (OK, the first condition is given for you. But what else must be true about the a_n ?)

- (i) The series is alternating.
- (ii)